

MUSIC for Faraday Rotation Measure Synthesis

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Accepted 1988 December 15. Received 1988 December 14; in original form 2012 October 15

ABSTRACT

Faraday Rotation Measure (RM) synthesis requires the recovery of the Faraday Dispersion Function (FDF) from measurements restricted to limited wavelength ranges, which is an ill-conditioned deconvolution problem. Here, we propose a novel deconvolution method based on an extension of the MULTiple Signal Classification (MUSIC) algorithm. The complexity and speed of the method is determined by the eigen-decomposition of the covariance matrix of the observed polarizations. We show numerically that for high to moderate Signal to Noise (S/N) cases the RM-MUSIC method is able to recover the Faraday depth values of closely spaced pairs of thin RM components, even in situations where the peak response of the FDF is outside of the RM range between the two input RM components. This result is particularly important because the standard deconvolution approach based on RM-CLEAN fails systematically in such situations, due to its greedy mechanism used to extract the RM components. For low S/N situations, both the RM-MUSIC and RM-CLEAN methods provide similar results.

Key words: data analysis - Techniques: polarimetric - magnetic fields

1 INTRODUCTION

Faraday rotation is a physical phenomenon where the position angle of linearly polarized radiation propagating through a magneto-ionic medium is rotated as a function of frequency. Faraday Rotation Measure (RM) synthesis is an important method for analyzing multichannel polarized radio data, where multiple emitting regions are present along the single line of sight of the observations (Brentjens & de Bruyn 2005). In practice, the method requires the recovery of the Faraday Dispersion Function (FDF) from measurements restricted to limited wavelength ranges, which is an ill-conditioned deconvolution problem, raising important computational difficulties. At least four different approaches have been proposed so far to solve this problem. A first approach uses an adaptation of the CLEAN algorithm (Hogbom 1974) to the RM deconvolution (RM-CLEAN) (Heald et al. 2009). The second approach is wavelet-based, and assumes field symmetries in order to project the observed data onto $\lambda^2 < 0$ (Frick et al. 2010). The third approach relies on nonlinear model fitting (Farnsworth et al. 2011), while the fourth approach is based on the novel compressed sensing (CS) paradigm (Li et al. 2011; Andrecut et al. 2012). Whether these methods are successful or not in detecting the structure of the FDF depends on the Signal to Noise ratio (S/N), the separation of the RM components, the relative angle of the RM components

at the observed wavelengths, and the wavelength range. For example, the current existing methods fail to recover the correct RM components when the separation among them is below the resolution determined from the half maximum of the main peak of the RM spread function (RMSF). Also, another major problem is related to situations where the interference of pairs of RM components conspire to place the peak response of the FDF outside of the RM range between the two input RM components. In this case, the standard RM-CLEAN method fails, due to its intrinsic greedy mechanism used to pick up the RM components. Here we discuss a novel approach, which addresses these two significant problems from a different perspective. The proposed method is an extension of the MULTiple Signal Classification (MUSIC) algorithm, which is based on the eigen-decomposition (ED) of the covariance matrix of the observed data (Cheney 2001). The complexity and the speed of the method is therefore determined by the size of the eigen-decomposition problem, which for several hundreds of data points is comparable to the RM-CLEAN approach. Our numerical results show that for high to moderate S/N cases, the RM-MUSIC method gives very good results, outperforming the standard approach based on RM-CLEAN. For low S/N situations, both the RM-MUSIC and RM-CLEAN methods provide similar results. We should note that contrary to the other existing methods, the RM-MUSIC method recovers only the Faraday depth values. Once the Faraday depth values are determined, the real and imaginary parts of the RM components

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can be easily computed using the linear least squares fitting approach.

2 RM-SYNTHESIS

The Faraday depth (in rad m^{-2}) is defined as:

$$\phi(r) = 0.81 \int_{\text{source}}^{\text{observer}} n_e \mathbf{B} \cdot d\mathbf{r}, \quad (1)$$

where n_e is the electron density (in cm^{-3}), B is the magnetic field (in μG), and dr is the infinitesimal path length (in parsecs). We also define the complex polarization as:

$$P(\lambda^2) = Q(\lambda^2) + iU(\lambda^2) = pIe^{2i\chi(\lambda^2)}, \quad (2)$$

where p is the fractional polarization, I , Q , U are the observed Stokes parameters, and $\chi(\lambda^2)$ is the polarization angle observed at wavelength λ . Also, we assume that the observed polarization $P(\lambda^2)$ originates from the emission at all possible values of ϕ , corresponding to the Fourier transform:

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi, \quad (3)$$

where $F(\phi)$ is the complex FDF (the intrinsic polarized flux, as a function of the Faraday depth). Thus, in principle $F(\phi)$ is the inverse Fourier transform of the observed quantity $P(\lambda^2)$:

$$F(\phi) = \int_{-\infty}^{+\infty} P(\lambda^2) e^{-2i\phi\lambda^2} d\lambda^2. \quad (4)$$

However, this operation is ill-defined since we cannot observe $P(\lambda^2)$ for $\lambda^2 < 0$, and also in general the observations are limited to an interval $[\lambda_{min}^2, \lambda_{max}^2]$.

In order to deal with the above limitations, the observed polarization is defined as:

$$\tilde{P}(\lambda^2) = W(\lambda^2)P(\lambda^2), \quad (5)$$

where W is the observation window function, with $W(\lambda^2) > 0$ for $\lambda^2 \in [\lambda_{min}^2, \lambda_{max}^2]$, and $W(\lambda^2) = 0$ otherwise. Therefore, for N measurement channels with frequencies ν_n , wavelengths $\lambda_n = c/\nu_n$, $n = 1, 2, \dots, N$ (c is the speed of light), weights $W(\lambda_n^2) = W_n$, polarizations $P(\lambda_n^2) = P_n$, we obtain the following discrete expression for the FDF:

$$F(\phi) = A^{-1} \sum_{n=0}^{N-1} W_n P_n e^{-2i\phi(\lambda_n^2 - \lambda_r^2)}, \quad (6)$$

where

$$A = \left[\sum_{n=0}^{N-1} W_n \right]^{-1} \quad (7)$$

is a normalization constant, and the reference wavelength λ_r^2 is defined as:

$$\lambda_r^2 = A^{-1} \sum_{n=0}^{N-1} W_n \lambda_n^2. \quad (8)$$

In our approach we assume that the model of $F(\phi)$ contains $K \ll N$ (unknown) components $f_k \delta(\phi - \phi_k)$, where f_k are complex and ϕ_k are real quantities $k = 0, \dots, K-1$:

$$F(\phi) = \sum_{k=0}^{K-1} f_k \delta(\phi - \phi_k). \quad (9)$$

Considering additive noise in the measurement process, the sampled polarization-domain channel response is given by:

$$P_n = W_n \sum_{k=1}^K f_k e^{2i\phi_k(\lambda_n^2 - \lambda_r^2)} + \alpha_n + i\beta_n, \quad n = 1, 2, \dots, N, \quad (10)$$

where N is the number of measured channels. Here, both α_n and β_n are Gaussian variables with mean zero and standard deviation σ . Obviously, $F(\phi)$ is a “dirty” reconstruction, and a deconvolution step is necessary to recover the RM components $f_k \delta(\phi - \phi_k)$, given the observed values P_n and the weights W_n .

3 RM-MUSIC

The MUSIC method is generally used for estimating frequencies in signal processing problems, and the location of pointlike scatterers in imaging problems (see Cheney (2001) for a review). The standard MUSIC method is based on the ED of an Hermitian operator, which corresponds to the covariance matrix of the signal or the array response matrix. By the finite-dimensional spectral theorem, such operators can be associated with an orthonormal basis of the underlying space in which the operator is represented as a diagonal matrix with real number entries. The main idea is to estimate the frequencies, or to localize multiple sources, by exploiting the eigen-structure of this Hermitian operator. More exactly the space spanned by its eigenvectors can be partitioned into two orthogonal subspaces, namely the signal subspace and the noise subspace. By exploiting the orthogonality of the signal and noise subspaces, the MUSIC method significantly improves the resolution (i.e. locating closely spaced frequencies or scatterers), and as a consequence it is considered a super-resolution method. Let us now formulate the standard MUSIC approach to the Faraday depth recovery problem.

Since only one snapshot of measurement data P of length N is available, the data sequence is divided into $M = N - L$ segments of length L , $0 < L \leq N$, and then the $L \times L$ covariance matrix is estimated as:

$$\hat{\mathbf{G}} = \frac{1}{M} \sum_{m=0}^{M-1} \mathbf{P}^{(m)} (\mathbf{P}^{(m)})^H \quad (11)$$

where $\mathbf{P}^{(m)} = [P_m, P_{m+1}, \dots, P_{m+L}]^T$, $m = 0, 1, \dots, M-1$, and the superscript H denotes the conjugate transpose operation. Suppose that $\mu_0 \geq \mu_1 \geq \dots \geq \mu_{L-1}$ and $\mathbf{g}^{(0)}, \mathbf{g}^{(1)}, \dots, \mathbf{g}^{(L-1)}$ are the eigenvalues, and respectively the eigenvectors of $\hat{\mathbf{G}}$, such that:

$$\hat{\mathbf{G}} \mathbf{g}^{(\ell)} = \mu_\ell \mathbf{g}^{(\ell)}, \quad (12)$$

and

$$\hat{\mathbf{G}} = \sum_{\ell=0}^{L-1} \mu_\ell \mathbf{g}^{(\ell)} (\mathbf{g}^{(\ell)})^H. \quad (13)$$

Since the measured signal contains only K components, the last $L - K$ eigenvalues of $\hat{\mathbf{G}}$, $\mu_K \geq \mu_{K+1} \geq \dots \geq \mu_{L-1}$, should be small and below the noise level, and we say that

the corresponding eigenvectors $\mathbf{g}^{(K)}, \mathbf{g}^{(K+1)}, \dots, \mathbf{g}^{(L-1)}$ span the noise subspace of $\hat{\mathbf{G}}$, while the first K eigenvectors $\mathbf{g}^{(0)}, \mathbf{g}^{(1)}, \dots, \mathbf{g}^{(K-1)}$ span the signal subspace, and the corresponding eigenvalues and eigenvalues $\mu_0 \geq \mu_1 \geq \dots \geq \mu_{K-1}$ should be above the noise level. Thus, the eigen-decomposition of $\hat{\mathbf{G}}$ can be written as:

$$\hat{\mathbf{G}} = \underbrace{\sum_{\ell=0}^{K-1} \mu_{\ell} \mathbf{g}^{(\ell)} (\mathbf{g}^{(\ell)})^H}_{\text{signal subspace}} + \underbrace{\sum_{\ell=K}^{L-1} \mu_{\ell} \mathbf{g}^{(\ell)} (\mathbf{g}^{(\ell)})^H}_{\text{noise subspace}}. \quad (14)$$

We can now form the projection operator onto the noise subspace defined as:

$$\hat{\mathbf{G}}_{\mathbf{g}}^{(\text{noise})} = \sum_{\ell=K}^{L-1} \mathbf{g}^{(\ell)} (\mathbf{g}^{(\ell)})^H, \quad (15)$$

and consider the signal subspace sampling vector

$$\mathbf{h}(\tilde{\phi}) = [e^{2i\tilde{\phi}\tilde{\lambda}_0^2}, e^{2i\tilde{\phi}\tilde{\lambda}_1^2}, \dots, e^{2i\tilde{\phi}\tilde{\lambda}_{L-1}^2}]^T, \quad (16)$$

where the wavelengths are interpolated as following:

$$\tilde{\lambda}_{\ell}^2 = \left[\frac{c}{\nu_N - \frac{\ell}{L-1}(\nu_N - \nu_1)} \right]^2, \quad \ell = 0, 1, \dots, L-1, \quad (17)$$

such that they cover the initial observation interval $[\lambda_{\min}, \lambda_{\max}]$. We should note that since $L \leq N$, the Faraday depth is also linearly interpolated as $\tilde{\phi} = \phi L/N$, and therefore we obtain:

$$\mathbf{h}(\phi) = [e^{2i\frac{L}{N}\phi\tilde{\lambda}_0^2}, e^{2i\frac{L}{N}\phi\tilde{\lambda}_1^2}, \dots, e^{2i\frac{L}{N}\phi\tilde{\lambda}_{L-1}^2}]^T. \quad (18)$$

If the vector $\mathbf{h}(\phi)$ represents the signal, i.e. it is a linear combination of the signal subspace eigenvectors, then its projection onto the noise subspace must be close to zero:

$$\|\hat{\mathbf{G}}_{\mathbf{g}}^{(\text{noise})} \mathbf{h}(\phi)\| \simeq 0. \quad (19)$$

Thus, the Faraday depth values $\phi_k, k = 0, 1, \dots, K-1$, should correspond to the maxima of the following MUSIC pseudo-spectrum:

$$S(\phi) = \frac{1}{\|\hat{\mathbf{G}}_{\mathbf{g}}^{(\text{noise})} \mathbf{h}(\phi)\|^2} = \frac{1}{\sum_{\ell=K}^{L-1} \|(\mathbf{g}^{(\ell)})^H \mathbf{h}(\phi)\|^2}. \quad (20)$$

Once the Faraday depth values ϕ_k are determined from the MUSIC pseudo-spectrum the real and imaginary parts of the RM components f_k can be easily computed using the linear least squares fitting approach, i.e. by solving the following minimization problem:

$$\begin{aligned} \{f_k\}_{k=0}^{K-1} &= \arg \min_{f_k} \sum_{n=0}^{N-1} \left[P_n - \sum_{k=1}^K f_k e^{2i\phi_k \lambda_n^2} \right]^2 \\ &= \arg \min_{\mathbf{f}} \|\mathbf{P} - \hat{\Psi} \mathbf{f}\|. \end{aligned} \quad (21)$$

This least-squares fit problem has fewer free parameters, and it is far better constrained than the fit that would have been done without MUSIC. In fact the problem has an unique minimum norm solution, obtained by solving the linear system of equations: $\hat{\Psi} \mathbf{f} = \mathbf{P}$, where $\mathbf{f} = [f_0, f_1, \dots, f_{K-1}]^T$ are the unknown components, $\mathbf{P} = [P_0, P_1, \dots, P_{N-1}]^T$ are the observed polarizations, and $\hat{\Psi}$ is the Fourier matrix with the elements $\Psi_{n,k} = e^{2i\phi_k \lambda_n^2}$. Thus, the least squares solution is given by $\mathbf{f} = \hat{\Psi}^\dagger \mathbf{P}$, where $\hat{\Psi}^\dagger = (\hat{\Psi}^H \hat{\Psi})^{-1} \hat{\Psi}^H$ is the Moore-Penrose pseudo-inverse of $\hat{\Psi}$.

Algorithm 1 Pseudo code of the RM-MUSIC method.

\mathbf{P} ; polarization data

N ; number of channels

$N/2 \leq L \leq N$; length of data segments

$M = N - L$; number of data segments

$\hat{\mathbf{G}} \leftarrow \frac{1}{M} \sum_{m=0}^{M-1} \mathbf{P}^{(m)} (\mathbf{P}^{(m)})^H$; covariance matrix

$\{\mu_{\ell}, \mathbf{g}^{(\ell)}\}_{\ell=0}^L \leftarrow ED(\hat{\mathbf{G}})$; eigen-decomposition

$\{\tilde{\lambda}_{\ell}^2\}_{\ell=0}^L \leftarrow \left[\frac{c}{\nu_N - \frac{\ell}{L-1}(\nu_N - \nu_1)} \right]^2$; interpolation wavelengths

$\mathbf{h}(\phi) \leftarrow [e^{2i\frac{L}{N}\phi\tilde{\lambda}_0^2}, e^{2i\frac{L}{N}\phi\tilde{\lambda}_1^2}, \dots, e^{2i\frac{L}{N}\phi\tilde{\lambda}_{L-1}^2}]^T$; sampling vector

$K \leftarrow \arg \min_K H(K)$; number of components

$S(\phi) \leftarrow \frac{1}{\sum_{\ell=K}^{L-1} \mu_{\ell}^{-1} \|(\mathbf{g}^{(\ell)})^H \mathbf{h}(\phi)\|^2}$; MUSIC pseudo-spectrum

$\{\phi_k\}_{k=0}^{K-1} = \{\arg \max_{\phi} S(\phi)\}_{k=0}^{K-1}$; Faraday depths

$\{f_k\}_{k=0}^{K-1} = \arg \min_{\mathbf{f}} \|\mathbf{P} - \hat{\Psi} \mathbf{f}\|$; complex amplitudes

return $\{\phi_k, f_k\}_{k=0}^{K-1}$;

4 NUMERICAL IMPLEMENTATION

In order to optimally use the MUSIC method we also need to determine the number of components K in the polarization signal. In principle, K can be determined from the eigenvalues of the covariance matrix. The first largest eigenvalues, which are much bigger than the noise level $\mu_{\ell} > \sigma^2$, $\ell = 0, 1, \dots, K-1$, should indicate the value of K . However, in a practical implementation, when the covariance matrix is estimated from a small number of observations, it is challenging to clearly separate the signal eigenvalues from the noise eigenvalues. In these ambiguous cases one can use information theoretic criteria for selection, like the Minimum Descriptive Length (MDL) criteria, where the value of K corresponds to the minimum of the following quantity:

$$\begin{aligned} H(K) &= -M(M-K) \log \left(\frac{\prod_{m=K}^M \mu_m^{1/(M-K)}}{\frac{1}{M-K} \sum_{m=K}^M \mu_m} \right) \\ &+ \frac{1}{2} K(2L-K) \log M. \end{aligned} \quad (22)$$

Also, since the smallest eigenvalues are not equal among them, one can use the following normalized versions of the MUSIC pseudo-spectrum:

$$S(\phi) = \frac{1}{\sum_{\ell=K}^{L-1} \mu_{\ell}^{-1} \|(\mathbf{g}^{(\ell)})^H \mathbf{h}(\phi)\|^2}, \quad (23)$$

that accounts for the variation of the eigenvalues, and it is less sensitive to the K estimation errors. Another influencing parameter is the length of the data segments L . In order to preserve as much information as possible in each data segment we should have $L \geq N/2$. Our numerical experiments have shown that $L = 2N/3$ has a good detection sensitivity, and this is the value used in the simulations presented here. We should note also that the speed of the RM-MUSIC depends on the parameter L , which gives the size $L \times L$ of the eigen-decomposition problem. For L in the order of hundreds, the speed of RM-MUSIC is comparable to the speed of RM-CLEAN, or even faster. The pseudo-code of the RM-MUSIC method is given in Algorithm 1.

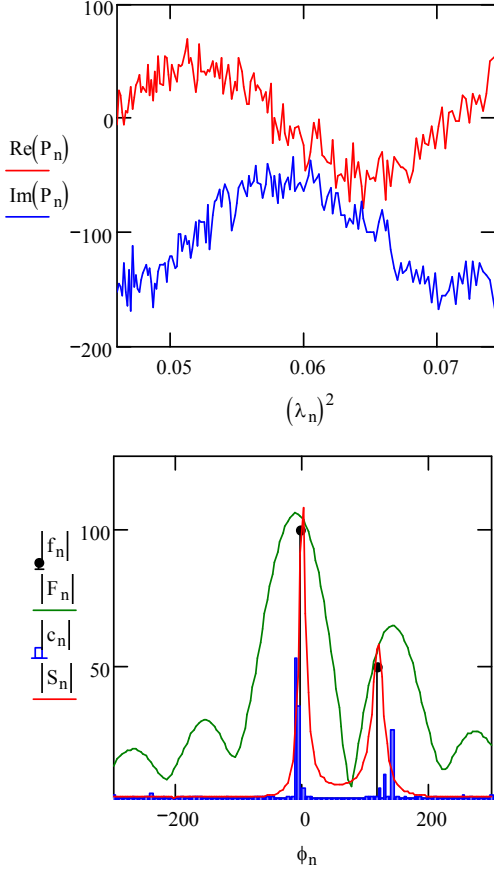


Figure 1. RM-MUSIC successful result for two components with the complex amplitudes $f_1 = -100i$, $f_1 = 50i$, and Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 122 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_2 \simeq \delta\phi$ ($|f|$ component amplitudes, $|F|$ “dirty” FDF, $|c|$ RM-CLEAN components, S RM-MUSIC spectrum).

5 NUMERICAL RESULTS

To illustrate numerically the RM-MUSIC method we consider the following experiment layout: frequency range: $\nu_{min} = 1100 \text{ MHz}$, $\nu_{max} = 1400 \text{ MHz}$; wave length range $\lambda_{min}^2 = 0.046 \text{ m}^2$, $\lambda_{max}^2 = 0.074 \text{ m}^2$; number of observation channels $N = 150$; width of an observing channel $\Delta\nu = 2 \text{ MHz}$; resolution in Faraday depth space $\delta\phi = 2\sqrt{3}/(\lambda_{max}^2 - \lambda_{min}^2) \simeq 122 \text{ rad m}^{-2}$; equal weights $W_n = 1$; noise level $\sigma = \sqrt{N} = 12.247$.

The RM-MUSIC method works very well when the components are separated above the resolution limit $\delta\phi$. For example, in Figure 1 we have two components with the complex amplitudes $f_0 = -100i$ and $f_1 = 50i$ at $\lambda^2 = \lambda_n^2$, and Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 122 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_0 = \delta\phi$. For comparison we also give the “dirty” FDF and the RM-CLEAN components. In order to make the results comparable, we scaled $S(\phi)$ as $S(\phi) \leftarrow S(\phi) |F|_{max} |S|_{max}^{-1}$, where $|F|_{max}$ and $|S|_{max}$ are the maximum amplitudes of the original spectra. This way, both $S(\phi)$ and $|F(\phi)|$ are in the same range. One can see that the RM-MUSIC spectrum recovers almost exactly the Faraday depths of the two components $\tilde{\phi}_0 = 1.05 \text{ rad m}^{-2}$, $\tilde{\phi}_1 = 122.81 \text{ rad m}^{-2}$, while both the “dirty” FDF and RM-CLEAN fail to recover the Faraday depth of the second component: $\tilde{\phi}_0 = 2.01 \text{ rad m}^{-2}$, $\tilde{\phi}_1 = 142.95 \text{ rad m}^{-2}$. Moreover,

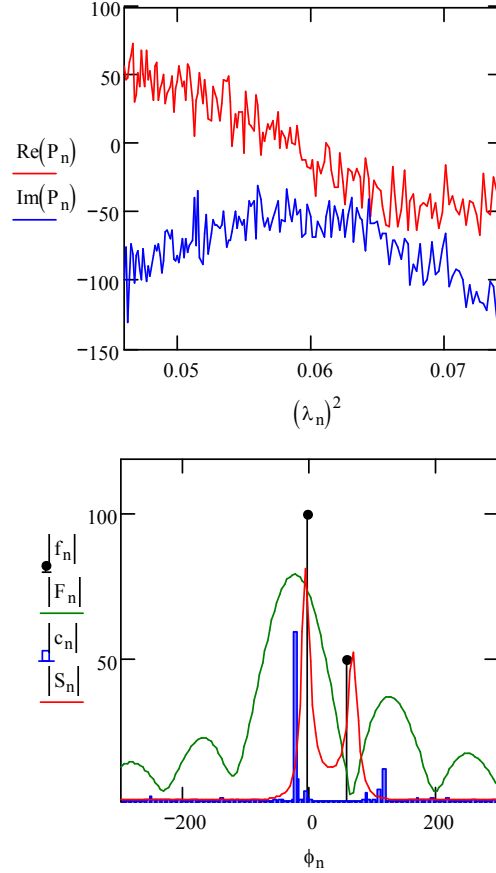


Figure 2. RM-MUSIC successful result for two components with the complex amplitudes $f_1 = -100i$, $f_1 = 50i$, and Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 61 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_2 \simeq 0.5\delta\phi$ ($|f|$ component amplitudes, $|F|$ “dirty” FDF, $|c|$ RM-CLEAN components, S RM-MUSIC spectrum).

once the depths are recovered, the real and imaginary parts of the RM components can be easily computed using a linear least squares fitting approach, resulting in an almost exact recovery: $\tilde{f}_0 = -1.95 - 99.74i$, $\tilde{f}_1 = 0.05 + 50.02i$.

In Figure 2 we show that the MUSIC super-resolution method can give good results even for separations below the resolution in Faraday depth space $\delta\phi$. Here, we have the same two components with Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 61 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_0 = 0.5\delta\phi$. One can see that both components are correctly separated by the RM-MUSIC spectrum $\tilde{\phi}_0 = 0 \text{ rad m}^{-2}$, $\tilde{f}_0 = 0.84 - 100.93i$, $\tilde{\phi}_1 = 62.4 \text{ rad m}^{-2}$, $\tilde{f}_1 = -0.76 + 51.49i$. and once again the “dirty” FDF and RM-CLEAN fail to recover the Faraday depths, showing two main components at: $\tilde{\phi}_0 = -6.4 \text{ rad m}^{-2}$, and respectively $\tilde{\phi}_1 = 118.8 \text{ rad m}^{-2}$. This is a typical example where the interference of pairs of RM components conspire to place the peak response of the FDF outside of the RM range between the two input RM components. In Figure 3 we consider the same two components separated by $\phi_1 - \phi_0 \simeq 0.25\delta\phi$, $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 30.5 \text{ rad m}^{-2}$. In this case the RM-MUSIC spectrum cannot separate the components anymore, however it correctly shows only one maximum, situated between the two input RM components, while RM-CLEAN suggests a single RM component outside the range of the input RMs.

From the above examples we see that RM-MUSIC per-

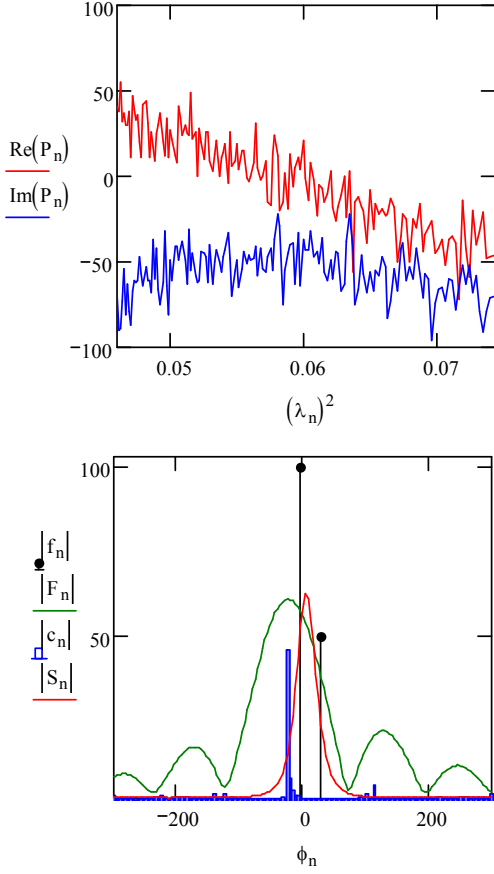


Figure 3. RM-MUSIC result for two components with the complex amplitudes $f_1 = -100i$, $f_1 = 50i$, and Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 30.5 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_0 \simeq 0.25\delta\phi$ ($|f|$ component amplitudes, $|F|$ “dirty” FDF, $|c|$ RM-CLEAN components, S RM-MUSIC spectrum).

forms better than RM-CLEAN for high to moderate S/N cases. However, in low S/N situations the performance of RM-MUSIC deteriorates. It is quite difficult to quantify the S/N ratio exactly and to find a limiting S/N value, and more future evaluations of the method are required for various wavelength ranges and noise levels per channel. For the considered experiment layout our numerical simulations have shown that if the amplitude of the strongest RM component is becoming smaller than $|f_0| \simeq 50$, and for a noise per channel $\sigma = \sqrt{N} = 12.247$, RM-MUSIC begins to behave more like RM-CLEAN, and cannot separate correctly the input components. In Figure 4 we give such an example of two components with the complex amplitudes $f_0 = -30i$ and $f_1 = 15i$, Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$ and $\phi_1 = 61 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_0 \simeq 0.5\delta\phi$. One can see that in this case, both RM-MUSIC and RM-CLEAN give similar results. We should also mention that the experiment layout used in simulations is very similar to the ASKAP’s POSSUM survey (Gaensler et al. 2010), where the expected final noise is $10 \mu\text{Jy/beam}$. This value suggests that for $N = 150$ channels and a peak signal to noise threshold of ~ 4 , RM-MUSIC should work as indicated for sources with polarized flux higher than 0.5 mJy/beam , which is rather bright but quite reasonable, since about $\sim 57\%$ of the NVSS RM catalog sources fall into this category.

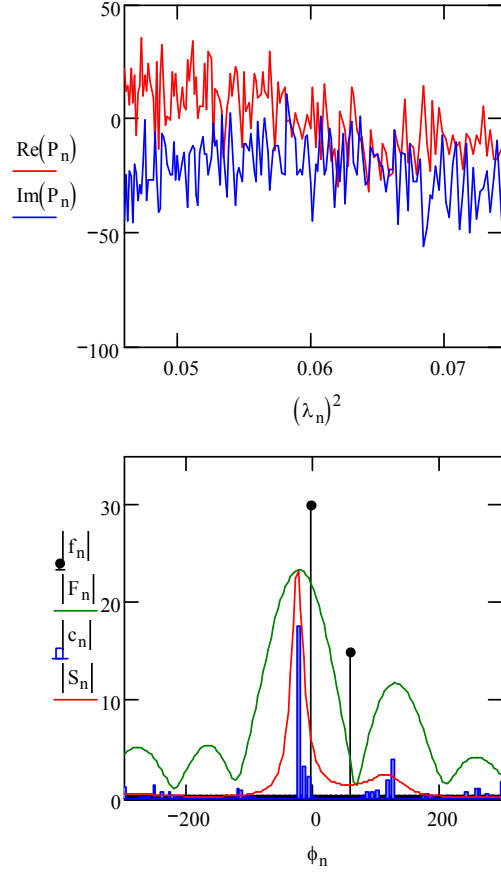


Figure 4. RM-MUSIC result for low signal to noise is comparable to RM-CLEAN. Two components with $f_1 = -100i$, $f_1 = 50i$, Faraday depths $\phi_0 = 0 \text{ rad m}^{-2}$, $\phi_1 = 61 \text{ rad m}^{-2}$, separated by $\phi_1 - \phi_0 \simeq 0.5\delta\phi$ ($|f|$ component amplitudes, $|F|$ “dirty” FDF, $|c|$ RM-CLEAN components, S RM-MUSIC spectrum).

6 CONCLUSION

We have discussed a novel deconvolution method for RM synthesis applications, based on an extension of the MUSIC super-resolution algorithm. Numerical results show that for high to moderate S/N cases RM-MUSIC outperforms the standard RM-CLEAN approach, being able to recover Faraday depth values of closely spaced pairs of thin RM components, even in situations where the peak response of FDF is outside the range between the two input components. For low S/N cases, the RM-MUSIC and RM-CLEAN methods provide similar results. RM-MUSIC performs well for pairs of thin RM components, however it is not suitable when the input Faraday spectrum contains polarized emission at a continuous range of RM.

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